Whitehead's Poetical Mathematics

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Θάλασσα διαχέεται καὶ μετρέεται εἰζ τὸν αὐτὸν λόγον ὀκοῖοζ πρόσθεν ἦν ἢ γενέσθαι γῆ

(The earth melts into the sea as the sea sinks into the earth).

Heraclitus[1]

In this essay, I trace a set of math-poetic figures from Whitehead's *Process and Reality* in order to understand how he constructs a theory of the world that prehends, feels, and becomes social. My essay centers on two principal questions: How does Whitehead construct a philosophy of process and organism on mathematical intuitions that retains nonetheless all the living qualities of the unbifurcated world? And to what degree and in what manner does he construct his pata-mathematical concepts the way a mathematician constructs and fabricates concepts? Given his philosophical and theological inheritance, Whitehead responds simply and remarkably to some of the most provocative mathematics and mathematical physics of his day: Bertrand Russell's and David Cantor's set theory, and Einstein's general relativity. But if he seems to respond too bluntly in some respects, to what philosophical purposes—not scientific or mathematical—does he set his speculation? In the latter part of this essay, I will try to extend Whitehead's speculation using "lures for feeling" made from measure theory and topological dynamical systems, and will

1. Heraclitus, *Fragments: The Collected Wisdom of Heraclitus*, trans. Brooks Haxton (New York: Viking, 2001), Fragment 23.

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outline a notion of process that does not appeal to objects.[2] I argue that elaborating Whitehead's speculation a few steps beyond his artfully blunted set theory and general relativity theory yields a way out of the static and atomistic aspects of his metaphysics. Indeed, my amicus curiae should substantially enrich a plenist and process-oriented concept of unbifurcated nature that more readily accommodates local novelty.

I came to Whitehead after thinking with Gilles Deleuze and Félix Guattari's multiplicity, Deleuze's appropriation of Riemannian manifolds, and the a-signifying semiology of Guattari's chaosmosis. I assure you that my reading is not some truth-seeking missile, but a speculative and poetic exercise in thinking through Whitehead's philosophy of process—with the alchemical accompaniment of all those nonhuman, mathematical objects, like the monsters of set theory (pace Alain Badiou), and the point-free topologies of René Thom and Alexander Grothendieck, whose more fertile philosophical consequences have hardly been adequately developed, I believe.

In their Heraclitus seminar, Martin Heidegger and Eugene Fink tried to steer a middle course between a close, closed hermeneutic study of Heraclitus's Fragments and a free-associative "philosophizing" with the putative sense of the Greek text.[3] With their fellow readers, they used the texts to develop a process theory that honored what they found in Heraclitus but also extended their phenomenological investigation. It seems worthwhile to read Whitehead in an analogous constructive and productive spirit to develop a topological approach to a process world.

First let me rapidly rehearse Whitehead's ontology as he develops it. In retracing *Process and Reality*'s argument, we can detect, albeit faintly, what a mathematician might recognize as the rhetoric of proof. These features include preliminary motivations established as

2. Alfred North Whitehead, *Process and Reality: An Essay in Cosmology*, Corrected ed., ed. David Ray Griffin and Donald W. Sherburne (New York: Free Press, 1978), p. 85. I borrow the expression in sympathy with a "propositional, (and not conversational)" approach to a processual mode of discourse proposed by Isabelle Stengers, who writes: "I take words to be 'lures for feeling,' not denouncing them because they would demarcate, . . . but demanding that those words would be constructed with the aims of 'clothing the dry bones,' . . . of our demarcations with the vivid feeling of the presence of those [who are not able to speak to the conversation because they are not inside the privileged domain of discourse] our demarcations cannot help but push away. Away but not against" (Isabelle Stengers, "Beyond Conversation," in *Process and Difference: Between Cosmological and Poststructuralist Postmodernisms*, ed. Catherine Keller and Anne Danielle [Albany: State University of New York, 2002], p. 238).

3. Martin Heidegger, and Eugen Fink, *Heraclitus Seminar* (Evanston, Ill.: Northwestern University Press, 1993), p. 6.

definitional "assumptions," paradigmatic examples, and a network of lemmas, theorems, and corollaries. He uses such labels almost nowhere because he supplies almost no arguments with the robustness and precision of a mathematical proof.[4] (There is no call for actual mathematical argument, of course; and in fact, despite the formal precedents of Spinoza's *Ethics,* Newton's *Principia*, and Russell and Whitehead's *Principia Mathematica*, such rigor probably would sink the speculative enterprise.) Whitehead deploys a surfeit of assumptions, rather than finding a minimal model. One can see a paradigmatic example of this in his extravagant development of abstractive sets, about which his assumptions run into the dozens.[5] One difficulty is that his conceptual edifice is a floating circle of co-constructive notions: actual entity, prehension, concrescence, nexus (society)—and later on, apparently still more abstractly, ovate sets, abstractive sets, strain, duration. But to an archaeologist of mathematics, elements of the mathematical physics of that era figure prominently in Whitehead's construction, and it seems fruitful to understand what philosophical juice he extracts from those elements.

Whitehead begins with the ontological principle: everything comes from somewhere; nothing comes from nowhere. Philosophy must start with the concrete, and explain abstraction, not the reverse. One cannot derive the concrete from the abstract or the ideal. And the concrete does not come to us already split into symbolic and material categories. Indeed, nature is unbifurcated: it is a single, plenist ontology in which matter, matters of fact, feelings, subjective experience, the experiencing subject, and experienced entities are deeply entangled. (One wonders whether they could be fused in a molecular if not fieldlike way.) Bifurcation would split the world into causal, objective nature and a perceived nature.[6] However, Whitehead insists that our experience of matter comes as actual entities,[7] because, as he put it, "continuity concerns what is potential; whereas actuality is incurably atomic."[8] The heart of his argument rests on an appeal to intuition and first impressions:

4. It would be interesting to compare Whitehead's speculative project with those of Edmund Husserl, Alain Badiou, René Thom, and Alain Connes that, however diverse, exhibit the same flair of mathematical, constructive imagination. Each of these was schooled in mathematics to a professional level; Thom and Connes contributed at the top level of their disciplines over a lifetime of sustained mathematical practice.

5. Whitehead, *Process* (above, n. 2), pp. 297–333.

6. Isabelle Stengers, "Whitehead's Account of the Sixth Day," in this issue

7. Whitehead, *Process* (above, n. 2), p. 214.

8. Ibid., p. 61.

> In their most primitive form of functioning, a sensum is felt physically with emotional enjoyment of its sheer individual essence. For example, red is felt with emotional enjoyment of its sheer redness. In this primitive prehension we have aboriginal physical feeling in which the subject feels itself as enjoying redness.[9]

Contra Hume, Whitehead posits that these aboriginal feelings spring up not from unknown causes but from actual occasions transmitting or conducting feelings vectorially to one another. And about the dynamics of actual occasions, he writes: "The sole appeal is to intuition."[10] He motivates the direct apprehension of actual entities as whole objects rather than as composites with his amusing observation that one dances with a whole human partner, not with a cloud of flickering sense data:

> A young man does not initiate his experience by dancing with impressions of sensation, and then proceed to conjecture a partner. His experience takes the converse route. . . . The true physical doctrine is that physical feelings are in their origin vectors.[11]

But this microversion of the anthropic principle, for that is what it is, gives the object-oriented argument a whiff of the tautological.[12]

Concrescence

For Whitehead, the becoming of an actual entity, akin to Heidegger's *anwesen*, is actually constitutive of that entity, so process, change, and even duration are intrinsically part of the raw material of his ontology. Indeed, this process of becoming is a basic, primitive element of his ontology: "[it] cannot be explained from higher order abstraction nor be broken into constituents."[13] Rather than a metaphysics or a theory of knowledge predicated on sense data, this is an account of a phenomenology based on embodied experience:

> For the organic theory, the most primitive perception is "feeling the body as functioning." This is a feeling of the world in the past; it is the inheritance of the world as a complex of feeling; namely, it is the feeling of derived feelings. The

9. Ibid., pp. 314–315 (emphases added).

10. More precisely, it is the concrescence of actual occasions that he has in mind when he writes: "The sole appeal is to intuition" (ibid., p. 22). But this comes to the same thing. See the discussion of concrescence below.

11. Ibid., pp. 315–316.

12. In answer to the question, why is the universe the way it is, the anthropic principle states: if the universe were much different, we would not be here to ask this question.

13. Whitehead, *Process* (above, n. 2), pp. 21–22.

> body, however, is only a peculiarly intimate bit of the world. Just as Descartes said, "this body is mine"; so he should have said, "this actual world is mine."[14]

Whitehead prepares the ground for an ontology that does not bifurcate between body and unfeeling world, or between local and global. His unbifurcated ontology is composed of occasionally infinite hierarchies of nested entities vivified by relations of feeling and sensing in time. This is materially where all the action lies. Using the notions of collectivity, "nexus," and "society," Whitehead tries to generate rich structure in the world in an unbifurcated way—but whether they are set-theoretic, or perhaps in a richer sense, category-theoretic notions, in any event they are built out of discrete entities, on points, rather than continua. Even in the limited (but infinite) world of mathematical logic and set theory, logicians face the essentially unavoidable technical difficulty of producing the continuum from a set of points.

Whitehead speaks of all actual entities (not just live vs. nonlive organisms) as having their concrete properties and characteristics reproduced in what he calls *prehensions*. Every character is reproduced in a prehension, and, most importantly, there is an indefinite number of prehensions. This indefiniteness yields a radically open metaphysics. A prehension is directional, hence has a "vector character," and, unlike raw sense data, it "involves emotion, and purpose, and valuation, and causation."[15] Via these prehensions, sensing, feeling, and pulling, the actual entities are engaged in the "production of novel togetherness," the coming together of many actual occasions into the novel actual occasion, a process that he calls *concrescence*.[16] He constructs a theory of time to suit this dynamic of the world using a notion of past, present, and future that does not rely on metrized, geometrized clock time, but on more elementary, topological notions of causal past, causal future, and the acausal complement in space-time. The acausal complement to an event is that part of the world whose occasions cannot affect or be affected by the event. Whitehead's causality is infinitely richer than the physicist's test of accessibility by light (along the geodesics with respect to the space-time manifold), but it formally parallels the logic of general relativity. This should be familiar to readers of Stephen Hawking and G. F. Ellis's classic *Large Scale Structure of Space-Time,* in which they demonstrated the expressive and explanatory power of Hawking's

14. Ibid., p. 81.

15. Ibid., p. 19.

16. Ibid., p. 21.

topological approach to space-time, accommodating even places of material infinity where the space-time metric and curvature become singular, infinite.[17] Topology articulates what exceeds number and metrized geometry.

I pass silently over Whitehead's construction of the "immediate" present, although it is one of the most important aspects of his treatment of time, and fold the discussion of time into a consideration of the larger dynamics of organism on which he rests his process theory. For the moment, we note that he tries to maintain his unbifurcated ontology by describing prehensions as feelings in time, and by associating groups of entities (*nexūs*) with societies—making the multiple social by propositional fiat. The atomicity of his actual occasions yields, in his concept of the world, a notion of time that is correspondingly atomic. However, any atomistic conception of the world that purports to account for the richer structures evident in our experience inevitably needs to posit dyadic and more generally n-adic relations between these atoms. So it is not surprising that Whitehead concludes that actual entities' concrete experience must be vectorial—that is, directed. We are not very far from Leibniz, except that Whitehead's monads are changeless and Whitehead's places are themselves immovable. This is a profound difference. If, by the ontological principle, everything actual is made of atomic, changeless, actual entities, how then can we account for change, and the potential for novelty and creativity?

In one of his most significant coercive neologisms, Whitehead makes actual entity synonymous with actual occasion.[18] It is important to recognize here, as Isabelle Stengers does, that Whitehead is making a propositional, conceptual identification, not a metaphorical one.[19] There are arguable and even demonstrable consequences

17. Stephen W. Hawking and George Francis Rayner Ellis, *The Large Scale Structure of Space-Time* (Cambridge: Cambridge University Press, 1973). Here, topology refers to the notion of domains as connected, open subsets of general (3,1) dimensional space-time, not graphs but open sets. I do not see on what philosophical grounds Whitehead claims that the past and future causal domains relative to an actual occasion must be disjoint. Indeed, early in the history of general relativity, Kurt Gödel discovered a cylindrical solution to the Einstein field equations that forced reconsideration of global vs. local causality: Kurt Gödel, "An Example of a New Type of Cosmological Solutions of Einstein's Field Equations of Gravitation," *Reviews of Modern Physics* 21 (1949): 447–450.

18. Whitehead, *Process* (above, n. 2), p. 22.

19. Remarking on Whitehead and Leibniz's method, Stengers writes: "the possibility of [what] . . . they wanted to construct, exhibits the creativity of mathematicians who

to such an identification. Stengers would say, rightly, that this is not a "merely" metaphorical identity, but it does seem that Whitehead deploys the term "occasion" to infuse temporality by connotation as well as fiat.

That being said, nonetheless, we can ask whether an equation like Whitehead's "actual entity = actual occasion" derives from and exerts different conceptual forces upon those who think it than does an identification like the tensor equation

$$G = 8\,\pi\,T$$

Figure 1. Einstein's equivalence between the metric tensor *G* and the stress-energy tensor *T*.

that articulates Einstein's equivalence principle. The tensor equation encodes and implies a large, conceptually definite set of algebraic symmetries and conditions that on one hand are invariant over multiple subjectivities, and on the other hand express a certain belief about ontology: that the geometrical structure of the world is *identical* with the dynamic distribution of matter-energy of the world, and that this identity works at the level of *dynamics*. By the way, the *geometry* encoded by the curvature tensor *G* in Einstein's equivalence is not the geometry of space, but the geometry of space+time, which includes the temporal in a single, unbifurcated, manifold. This sort of geometrization is *not* the geometry of Bergson's critique of geometrized time. (We will see later that Whitehead's measurement reverts to the geometry of spatial Euclidean space.) Whitehead's blunted version of Einstein's equivalence principle forgets this dynamics, I believe.

Whitehead uses the distribution of *strain* acting on what he calls *flat loci* to provide the dynamic for the world. We see the strong parallel between his technical description of strain and geometry in the latter part of *Process and Reality*, and the account of concrescence with which he starts the book. But this attempt to provide a dynamic is predicated on a curious reappropriation of force. Force is vectorial, and as such aptly traces the directional modes of consciousness, but it is not coherent to glue vectors of force to particu-

do not seek the solution *to*, but rather construct the possibility *of*, a solution to a problem. . . . When a mathematician produces a strange hypothesis, . . . it is not a matter of opinion [or description]. He or she has been constrained by the problem. . . . Concepts are required in the construction not of an opinion but of the possibility of a solution to a problem" ("Beyond Conversation" [above, n. 2, p. 242).

lar geometrical objects such as flat loci because generally they operate in different modalities of the plenum world.[20] To use a concrete example: the vectorial difference between a dog and a cat, however it is measured, is not a dog, or a cat, or any actual mammal. Moreover, a vectorial theory does not account for modes of consciousness that are not so directed as to be easily subsumed by a vectorial account of prehension and of concrescence. Whitehead himself describes this as the gradual objectification of vectorial experience into a "scalar" form, which is an inspired way to gloss what in phenomenological terms would be nonperspectival apperception.[21]

Dynamics

A strain yields movement resolving the strain. Whitehead identifies *duration* with *strain* (duration being a complete set of mutually contemporary actual occasions).[22] But this Aristotelian, or in terms of mathematical physics, zero-order[23] equivalencing introduces two mysteries, the first of which is mismatched physical dimensions: energy is force × distance or, more finely, the integral of force along a trajectory; and conversely, force is spatial difference of potential energy. On the other hand, however, duration is measured in time, which is not commensurate with the units (i.e., "dimensions") of force at all. This seems nonsensical, and so needs an argument at least as convincing as Einstein's argument for the equivalence principle.[24]

20. Using the more precise language of differential geometry, vectors live in the *tangent space* to the manifold in which Whitehead's flat loci live.

21. But prior to that, even at the stage of apperceiving the actual, concrete world, I would counterpose Walter Benjamin's or a poet's mode of distracted, undirected consciousness to the sort of directional consciousness that Whitehead takes as exemplary in constituting a vectorial dynamic in the world.

22. Whitehead, *Process* (above, n. 2), pp. 322–323. But Whitehead's use of completeness raises the specter of sets that, although their completion is by definition tamed by inclusion of the accumulation limits of all infinite sequences, are themselves sparse and pathological. For a thorough introduction to the relevant notions of completeness, accumulation, and limit, see Halsey L. Royden, *Real Analysis*, 3rd ed. (New York: Macmillan, 1988).

23. By order, I mean the degree of differencing, how many times a difference operator has been applied. Newton's force, for example, is proportional to the acceleration, which is the second difference—the difference of the difference of the displacement; therefore it is second order.

24. So long as we avoid formalism, a philosophical argument should attend to the philosophical consequences of some mathematical condition, theorem, or theory. Whitehead himself uses the calculus of limits to dismantle the philosophical force of Zeno's putative paradoxes. In this case, the physicists' "dimensional arguments" are a basic technique by which physicists code ontological consistency claims and check ontological consistency.

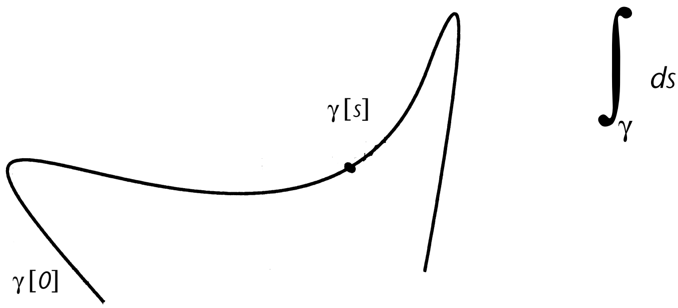

Figure 2. Integral along an arc γ parametrized by *s*.

The second mystery is that Whitehead gives no reason *why* such an equivalence should obtain, and he makes no observation about what that would afford the world. In fact, general relativity can be articulated in the same way via the language of differential geometry—that is, any local coordinate region in a space-time manifold is always automatically "fixed" and invariant with respect to time because, *by definition*, it carries itself. This is by definition a feature of all space-time geometries, Whiteheadian, Einsteinian, or otherwise. To elaborate, a space-time manifold is a topological space that locally has the geometry of a four-dimensional space with three spatial dimensions and a fourth dimension with the opposite signature. This is a local condition, so that what constitutes the "temporal direction" can vary continuously as we pass from event to event. This negative signature means that there is a qualitative difference between trajectories that flow temporally, and those that flow spatially, and those along which there is zero space-time metric displacement. The last sort of trajectory is exactly the set of geodesic paths traversed by light. The immotility of place characterizes any locus—any neighborhood of an event—on any such manifold, so Whitehead's argument for the fixity of actual occasions does not select his account over Einstein's general relativity.

Somewhat surprisingly, given his concern with process, Whitehead does not appeal to the calculus, in particular the differential, until very late in *Process and Reality*. There is a key moment where he argues against a purely geometric interpretation of a line element *ds* integrable to distance *s* along a curve, and proposes impetus (or impulse), which would intertwine matter and momentum.[25]

25. Whitehead, *Process* (above, n. 2), p. 323.

This intertwining echoes his 1922 book on relativity, in which he writes about "adjectival particles."[26] In an essay comparing Whitehead's and Einstein's theories of general relativity, Yutaka Tanaka perceptively says that Whitehead makes matter an "adjective" to spacetime.[27] While this is quite a suggestive expression, in more precise terms, Whitehead's most striking contribution is to insert a factor $J(s)$ into the line integral to form what one could interpret as an "impulse density" $J(s)\,ds$ along a trajectory. What if J were not a constant like "3" or "green," but a function varying according to the space-time locus and disposition?

$$\int_{\gamma} J[s]ds$$

Figure 3. Integral with impulse density.

But what we need is some insight into why this integral is important for Whitehead's project at all. That motivation comes from the variational, least-energy principle of dynamics. So what we should expect is some discussion of the philosophical adequacy of appealing to any variational principle whatsoever, because this underlies much of physics, and in this case the metaphysics.[28] I expect that Whitehead should disallow an apparently transcendentalist appeal to the principle of least action because it would contradict the ontological principle's injunction to start with the "concrete," which according to Whitehead is denominated in atomic, unchanging actual entities.[29] So, in order to intuit his derivation of dynamics we would need to comprehend the intuition behind getting kinetics (and more

26. A. N. Whitehead, *The Principle of Relativity with Applications to Physical Science* (Cambridge: Cambridge University Press, 1922), p. 34. I thank my reviewer for this reference.

27. Yutaka Tanaka, "Einstein and Whitehead: The Principle of Relativity Reconsidered," *Historia Scientiarum*, no. 32 (1987), pp. 45–61; *http://www.asahi-net.or.jp/~sn2y-tnk/tanaka_4_0.htm* .

28. In fact, David Hilbert used Emmy Noether's fundamental theorem to help derive the Einstein field equations from an action integral of the curvature of space-time.

29. For the same reason, Gilles Deleuze and Félix Guattari would avoid any such appeal as a motor to stir the magma of a thousand plateaus—as well as because of the principle's teleological nature. Localizing could partially address the latter concern insofar as local teleology does not imply global teleology.

generally, dynamics) from a potential field.[30] Force, which is directional, can be thought of as a spatial difference of potential energy, a field that to each position associates a directionless number. This is a *scalar* magnitude with no associated direction—in both Whitehead's and conventional mathematical usage. But since energy fields can be a function not only of locus but also of directional entities, that is, vectors (think of how much easier it is to swim with a prevailing wind than against it), we need some sort of measuring device that would yield a potential energy field from positional loci and directional entities. The simplest such machines are abstract algebraic functions called *tensors*, which provide a linear response to vectorial parameters. (And here I use "abstract" in the same sense as Deleuze and Guattari when they write about the abstract linguistic machine of language.) Analogically, linear response implies that doubling the prevailing wind doubles the energy expended to swim against it, and so forth.[31] We can obtain a *nonlinear* behavior by making nonlinear functions of the components of tensors. So, evaluating tensors on vector arguments at various loci yields a field of numerical values, a *scalar potential field*. If we imagine the states of a quasi-physical system evolving in time as particles on a scalar potential field, the least-energy variational principle derives change by having the particles sliding "downhill," from higher to lower states of scalar potential.

Measurement

Now, Whitehead needs to be able to measure his changeless, unmoving actual entities/occasions in order to feed them into his dynamical apparatus—his zero-order dynamics. And his Newtonian absoluteness will not allow him to resort to Einstein's moving clocks and meter sticks. In lieu of moving measuring devices, Whitehead offers a limiting process of fixed entities, traced by abstractive sets. But since what he wants to measure is any entity or *res vera*, he needs a more general sort of measure, and for that he appeals to a construction on sets that does not assume anything special about the metric, size, or geometry of what is being measured; in fact, he con-

30. The heart of Newton's calculus, captured in fact by the fundamental theorem of calculus, relates speed—the *derivative* of a function $F(x)$—with distance, the integral of a function $G(x)$. The derivation of dynamics from the spatial difference (called a *gradient*) of a potential field generalizes this fundamental relation between dynamics and potential field.

31. An operator T mapping a vector space to a vector space is said to be *linear* if it acts as follows on a linear combination of elements in its domain: for any x and y that are vectors in a vector space, and any scalar numbers a and b, $T[a{*}x + b{*}y] = a{*}T[x] + b{*}T[y]$.

structs *flat loci* that are defined prior even to the "spatial" and "temporal" categories. Whitehead's flat loci are generalizations of lines, more precisely of simplicial complexes, analogous to the vectors and multivectors that serve as arguments to ordinary tensors. Whitehead constructs his blunt version of lines from a simpler notion of extension, which for him is captured by the union and intersection of sets. To this end, he tries to build up "lines" as abstractive set limits of generalizations of planar ovals.

But if they are to measure any entity in the world, why should these model sets be two-dimensional? It is notorious how properties for ordinary shapes in two-dimensional Euclidean geometry can fail to extend to general sets in higher dimensions. For example, a continuous, closed loop in the plane separates the plane into two simply connected components, an inside and an outside. Finding a mathematically credible proof is surprisingly nontrivial, but perhaps not too surprisingly after one considers the topology of a Pollock. However, a continuous image of a two-dimensional sphere[32] may fail to separate three-dimensional Euclidean space into two simply connected components. Whitehead tries to generalize from ovals to "ovate sets" with analyzable intersection properties, from which he can build any linelike set as a limit of intersections (on the way back to vectorial experience). But why ovals? The intersection of two ovals is usually not an oval; so the set of ovals is not closed under the natural topological operation of intersection, and his attempt to generalize to "ovate" sets seems rather awkward and confusing.[33] The concept he is groping for is convexity, because the intersection of two convex sets is convex. The difficulties ensue from trying to force ovals and ovate sets into serving the general "measuring" purposes of a topological basis.

32. A sphere, for example—the locus of points given by $S = \{(x,y,z), x^2 + y^2 + z^2 = 1\}$, in three-dimensional Euclidean space, R^3—is a two-dimensional surface. The locus of points S happens to be a submanifold of R^3, but at any point on S, its neighborhood in S looks like (strictly speaking, is diffeomorphic to) a piece of the two-dimensional plane. Therefore it is a two-dimensional surface.

33. These flat loci anticipated topologies defined by embedded simplicial complexes. These topologies were proven in the 1950s, using category theory, to be functorially equivalent to topologies arising from cell complexes, and CW complexes. See, for example, William S. Massey, *A Basic Course in Algebraic Topology* (New York: Springer, 1997).Whitehead's nephew, John Henry Whitehead (1904–1960), was one of the most eminent founders of homotopy theory in topology and differential geometry. It would be interesting to discover what the elder Whitehead may have acquired from the younger, if anything. In fact, there is yet another mathematician-Whitehead: George Whitehead (1918–2004), who also worked in the field of algebraic topology, and who systematized algebraic topology via category theory.

With hindsight, we can see that Whitehead's project is weakened by an insistence on Euclid's ideal geometry of three-dimensional space. Topology would be more apt for his project because it articulates notions such as containment, boundary, point, density, intersection, union, and limit *without* appealing to number, measure, or even dimension. It is not geometry but topology that is "the investigation of the morphology of nexus."[34] Why limit the discussion to three-dimensional Euclidean space? In *Process and Reality*, entities are so general that there is no call for measuring them by three-dimensional sets at all. (Consider the set of all trajectories taken by the set of all people on Earth this year.) Aside from the reliance on Euclidean 3-space, there are three other challenges to Whitehead's approach to measurement.

Figure 4. An unbounded video stream is an example of a nonconvergent sequence.

The first challenge is a fundamental "linearity" in Whitehead's thinking about limits. In the *Concept of Nature*, where he tries to get at the points and linear subsets of (*Euclidean*) space as limits, as convergence to an "absolute minimum of intrinsic character"[35] via his abstractive sets, Whitehead makes an unexamined relation between geometric limits and analytic limits. Here I am using "analysis" in its technical mathematical sense.[36] Considering a sequence $e_1, e_2, e_3, \ldots$, with associated qualities $q(e_1), q(e_2), q(e_3), \ldots$, Whitehead claims that the associated values converge to a definite limit[37]—but this is in general not true, except for trivial sequences, like

$$\alpha, \alpha, \alpha, \alpha, \ldots .$$

In fact, the convergence of a sequence of elements in a topological space depends on the *topological*, not metric, properties of that am-

34. Whitehead, *Process* (above, n. 2), p. 302.

35. Alfred North Whitehead, *The Concept of Nature* (Amherst, N.Y.: Prometheus Books, 2004), pp. 85–86.

36. Throughout this essay, unless otherwise stated, I use "analysis" and "analytic" to refer to the mathematical study of the set of real numbers **R**, and of functions of a real variable. In this context, I also use "real" in its technical sense referring to the number system **R**, not as an adjective about ontological status.

37. Whitehead, *Concept of Nature* (above n. 35), p. 81.

bient space; for example, whether the ambient space is compact or sequentially compact. Therefore the integers, and metric geometry—in particular Euclidean, geometry—do not provide an adequate articulation of measure for Whitehead's dynamics.[38]

A second challenge is that Whitehead relies on sequential compactness where he could use a notion of compactness that does not rely on enumerable series, subject to the general, post-Pythagorean fixation with *counting* and *countability*. Roughly, sequential compactness is the phenomenon where an infinite sequence of elements or points in a topological space contains a limit point: an element near which an infinite number of its peers can be found, no matter how tightly one forms a neighborhood containing it. Quite understandably, Whitehead indexes infinite sequences of sets using the integers whereas by the transfinite Axiom of Choice it is also possible to index a sequence of sets from an arbitrary and uncountable index set.[39] But this a subtle fact. A topology includes countable unions and arbitrary, possibly uncountable, intersections. Countability is important because it makes a difference as to whether the intersection of a nontrivial descending chain of sets is null or not.[40] Whitehead characteristically overpowers the problem by piling on assumptions to define it away.[41]

Still a third challenge is that the construction of abstractive sets is a road mined with confusion, where in fact a *lattice*—not a graph-theoretic lattice, but the *set*-theoretic notion—could serve as a radically fertile alternative.[42] Lattice theory provides a way to work with

38. These discoveries were made in the same period that Whitehead turned to general relativity, but did not attract the same level of attention that Einstein's work did.

39. The Axiom of Choice is the following: Suppose A is a family of nonempty sets. Then there is always a function P defined on A such that for every set C belonging to the family A, $P(C)$ is an element of C. Note that the sets can be uncountably infinite, and the collection A can be arbitrary.

40. Counting figures largely also for another philosopher, Alain Badiou, who has made much use of contemporary set theory as a philosophical alternative to symbolic logic. In Badiou's theory, count-for-one, a process of identity formation that creates unity out of multiplicity, is an absolutely central act.

41. My assessment is a propositional and poetic remark, not a finding of matters of fact. In mathematics there are indeed no matters of fact.

42. Lattice set theory is not a pictorial theory of meshes, but a structural theory about sets and relations among sets built out of basic properties such as an order relation between sets. The most powerful aspect of this theory, for philosophical purposes, is that it is built entirely without referring to points or elements in sets. For example, subset and intersection are taken as basic operations without requiring us to "check" them by testing elements drawn from the sets. No "countable or uncountable" sequences of virtual moves are required, so in one stroke we eliminate the exhausting attempt to attain

sets without any reference to constituent points, starting only with the notion of partially ordering on sets $A \leq B$, and the binary operations "meet" and "join." These operations generalize the intersection and union of geometric subsets of Euclidean space. The most promising aspect of this approach to what mathematicians sometimes humorously call "pointless topology"—that is, the rigorous but nonrigid (or anexact, as Deleuze would put it) way to describe extended sets and substances without starting with atomic or point-like elements.

I trace Whitehead's process at this level of detail in order to show how he blunts mathematical process to serve his philosophical purpose, with artful but confusing results. Given all these challenges, what could a repotting of measurement in different mathematical soil offer Whitehead's account of process? Can we salvage measurement? Mathematical analysis's *measure theory* provides an alternative to understanding limits not merely as sequential limits, but as points of accumulation, or as intersections of infinite families of open sets indexed on some countable, or even more general transfinite, index set (like the field of ordered pairs of real numbers, or the set of square-integrable functions on the real line, which is infinite-dimensional).[43] Geometric measure theory approaches classical geometric entities like geodesics (locally length-minimizing curves in space-time) and tangent planes as limits of infinite processes in much larger, wilder, even monstrous spaces of mathematical structures.

Plenum and Process

Whitehead tries to honor a plenist spirit—explicitly acknowledging his debt to Spinoza—but I think he still commits in the end to an atomism: "Continuity concerns what is potential; whereas actuality is incurably atomic."[44] His ontology is dogged by a lack of access to the point-free topology constructible, for example, from lattice theory, measure theory, and differential geometry, some of which was developed after *Concept of Nature* and *Process and Reality* were published. On the other hand, Bernhard Riemann's fundamental invention of differentiable manifolds and Henri Poincaré's work

continuity or infinity by counting. A simple application of this is a foundation for a point-free (colloquially, "pointless") topology.

43. For introductions to square-integrable functions, to Hilbert spaces, and to real analysis—the study of the properties of the real line, of functions mapping the real line to itself, and extensions to sets in general—see Royden, *Real Analysis* (above, n. 22). For an introduction to geometric measure theory, see Frank Morgan, *Geometric Measure Theory: A Beginner's Guide*, 3rd ed. (San Diego: Academic Press, 2000).

44. Whitehead, *Process*, (above, n. 2) p. 61.

with dynamical systems had already been circulating for decades.[45] Of course, every such observation about a philosopher's conceptual (not metaphorical) use of mathematics should be tempered by the sympathetic understanding that speculation or improvisation requires a trellis, not a carapace, in order to sustain the imaginary. Therefore, alternative, nonaxiomatic constructions of ontology such as Whitehead's appropriation and blunting of the stress-energy tensor, and attempted invention of what later was systematized under point-free topology, light the way for further poetic exercises in the philosophy of process.

Whitehead also tries to honor the instinct toward process that characterizes an understanding of the world alternative to that of atoms and synchronic taxonomy. He is concerned, as he puts it, with the creativity "by which the many…become the one actual occasion" in a "production of novel togetherness" that he terms "concrescence."[46] But the basic dynamic, concrescence—this process in which many unities become integrated into a novel unity—being itself predicated on a discrete topology, is fundamentally as ideological a commitment as any metaphysics. It would be worthwhile seeing what ego-free *phenomenology* and languaging (instead of "I see a green tree," "the tree greens")[47] would be articulated by a topology that requires no points: it may be difficult to imagine, after decades of war declared by the logicians and engineers of graph theory, but far richer patterns than enumerated sets and graphs are ready to hand, such as Alexander Grothendieck's topos theory.[48] In any case, even short of such high-octane mathematics, Riemannian differen-

45. See, for example, Georg Friedrich Bernhard Riemann, "Habilitationsschrift: Ueber die Hypothesen, welche der Geometrie zu Grunde liegen" [1854], in *Gesammelte mathematische Werke und wissenschaftlicher*, ed. Heinrich Martin Weber, Nachlass, 2nd ed., (Leipzig: Teubner, 1892, Nendeln: Sändig Reprint, 1978).

46. Whitehead, *Process*, (above, n. 2) p. 21..

47. I thank Mick Halewood for inspiring this example in his talk "Becoming Actual—Whitehead and Deleuze on Subjectivity and Materiality," delivered at the Deleuze, Whitehead, and the Transformations of Metaphysics Symposium on May 24, 2005, at the Royal Flemish Academy in Brussels, Belgium.

48. Alexander Grothendieck's main work was published in *Éléments de géométrie algébrique* (Bures-sur-Yvette: Institut des Hautes Études Scientifiques, 1960-67). See also *http://www.math.jussieu.fr/~leila/mathtexts.php*. For a light introduction just to *topos*, see John Baez's web article "Topos Theory in a Nutshell," *http://math.ucr.edu/home/baez/topos.html* (January 27, 2004); and F. William Lawvere and Stephen H. Schanuel, *Conceptual Mathematics: A First Introduction to Categories* (Cambridge: Cambridge University Press, 1997).

tial geometry and Poincaré's dynamical systems, and topological dynamics from the perspective of geometry, furnish rich and suggestive handles for emergence, morphogenesis, and becoming that need no appeal to a differencing of discrete entities, or to discrete, computable algorithms.[49] (I have in mind the Russian geometric analyst of dynamical systems Vladimir Igorevich Arnold.)

Whitehead considers Heraclitus's "All things flow" (Fragment 41) and converts it to the vectorial impact of feelings of past entities upon an actual occasion being transformed into "scalar" feelings.[50] Mapping "All things flow" to "All things are vectors,"[51] he comes close to a process theory adequate to life, but not quite. Let me elaborate this process philosophy by taking three more speculative steps, just to see what might unfold. First, topology offers a way to articulate openness, neighborhood, and, most deeply, continuity without committing in advance to dimension, coordinate, degree of freedom, metric, or even finiteness. Who articulates this topological dynamic? I would say, with Deleuze, that it is not any subject or Subject, but rather, the world that articulates. Second, in a deep sense, acting in the mode of topology of *continua* obviates the recourse to counting, which is twin to number and discreteness. Articulations of continuity, and continuous articulations as developed by L.E.J. Brouwer in topology,[52] Heinz Hopf in global differential geometry, John Milnor in topology and analysis of manifolds, and William Meeks in minimal surfaces, exemplify some of these rich modes of material discourse.[53]

49. The consequences for this could be quite large. For example, we might still develop as rich an ontology as Whitehead's for Deleuze and Guattari's a-signifying semiology in chaosmosis without appealing to Deleuze's difference in itself. See Gilles Deleuze, *Logic of Sense*, p. 174, and as cited in Tim Clark's essay, "A Whiteheadian Chaosmosis?" in Keller and Daniell, *Process and Difference* (above, n. 2), p. 196.

50. Whitehead, *Process* (above, n. 2) p. 212.

51. Ibid., p. 309.

52. Luitzen Egbertus Jan Brouwer, *Collected Works: Geometry, Analysis, Topology and Mechanics*, ed. H. Freudenthal, vol. 2 (Amsterdam: North-Holland, 1975).

53. For a flavor of this sort of *geometrical* vs. analytic or algebraic modes of differential geometric reasoning, see, for example, the following canonical accounts: Heinz Hopf, *Differential Geometry in the Large: Seminar Lectures*, New York University, 1946 and Stanford University, 1956 (Berlin New York: Springer, 1983); John Willard Milnor, *Topology from the Differentiable Viewpoint* (Princeton, N.J.: Princeton University Press, 1997); and the article by William H. Meeks III, "Geometric Results in Classical Minimal Surface Theory," in *Surveys in Differential Geometry,* vol. 8 (Boston, Mass.: International Press, 2002), pp. 269–306.

Figure 5. Real-time, video resynthesis based on Navier-Strokes simulation of vorticular flow parametrized by live gesture.

Heraclitus wrote also,

Θάλασσα διαχέεται καὶ μετρέεται εἰζ τὸν αὐτὸν λόγον ὀκοῖοζ πρόσθεν ἦν ἢ γενέσθαι γῆ.

(the earth melts into the sea as the sea sinks into the earth). This field-based process of the world is *topological* as I have traced it in its simplest mode, that is, nonmetric and continuous. It is a poetic-philosophical figure of the earth and sea that neither reduces to counting points, nor inflates to complexity and chaos, but articulates richness. As I see it, and this is my third speculative step, measure theory's monsters and "pathologies" hint at an infinitely richer mathematical ontology ever more prolific than the present imaginary. The monstrous, in fact, occupies a region between the impossible and the potential real more fertile than that which Whitehead explicitly articulated, but that he, with Heraclitus, may have imagined.

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